understood that the overall heat transfer is more sensitive to deviations from boundary-layer theory than is the local heat transfer. Thus, the value of Gr_L at which the overall heat transfer begins to deviate from that of boundary-layer theory should be higher than the value of Gr_x at which deviations in the local heat transfer first occur. This reasoning is substantiated by comparing the value $Gr_x \sim 10^2$ from Table 2 ($Pr = 0.733$) with the aforementioned value $Gr_L \sim 10^4$ from experiment.

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TURBULENT SUBLAYER TEMPERATURE DISTRIBUTION INCLUDING WALL INJECTION AND DISSIPATION

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NOMENCLATURE

C_f, skin friction coefficient;
C_H, Stanton number;
k, conductivity;
Ec, Eckert number, $Ec = u_\infty^2/[gc_p(T_w - T_\infty)]$;
Pr, Prandtl number, $Pr = \mu c_p/k$;
Re, Reynolds number, $Re = u_\infty x/\nu$;
t, temperature fluctuation;
T, temperature;
u, v, w, velocity fluctuations;
U, V, W, mean velocity;
u^*, friction velocity, $u^* = \sqrt{(\tau_0/\rho)}$;
x, y, z, distance co-ordinates.

Greek symbols
μ, absolute viscosity;
ν, kinematic viscosity;
ρ, density;
τ, shear stress.

INTRODUCTION

FOR HEAT and mass transport at Prandtl and Schmidt numbers different than one the transfer rates are governed by the mechanisms which dominate near the wall. Most analyses depend on arbitrary or semi-empirical expressions for the distribution of the eddy diffusivity coefficient near the wall which do not even obey the governing differential equations.

Expressions have been derived for the assymptotic forms of momentum and heat transport near the wall by Tien and Wasan [1] and Tien [2]; while the effect of wall transpiration on velocity and momentum transport has been considered by Meroney [3]. This note extends the argument to the case of the thermal boundary layer with transpiration.

ANALYSIS NEAR WALL

For uniform turbulent flow over an infinite two dimensional flat plate of uniform wall temperature, changes of the flow variables in the x-direction are negligible near the wall compared to changes in the y-direction. Hence, the equations of motion and energy for a constant property incompressible flow may be written as

$$V_y = 0 \tag{1}$$

$$VU_y = \nu U_{yy} - (\overline{uv})_y \tag{2}$$

$$\rho c_p V T_y = k T_{yy} - (\rho c_p \overline{vt})_y + \mu\phi \tag{3}$$

where ϕ is the dissipation function and may be expressed as

$$\phi = \left(\frac{\partial U}{\partial y}\right)^2 + \frac{\partial^2 \overline{v}^2}{\partial y^2} + \left(\overline{\frac{\partial u}{\partial x}}\right)^2 + \left(\overline{\frac{\partial w}{\partial z}}\right)^2 + \left(\overline{\frac{\partial v}{\partial y}}\right)^2 + \left(\overline{\frac{\partial u}{\partial y}}\right)^2 + \left(\overline{\frac{\partial v}{\partial z}}\right)^2 + \left(\overline{\frac{\partial w}{\partial x}}\right)^2 + \left(\overline{\frac{\partial w}{\partial y}}\right)^2.$$

Near the wall the velocity and temperature terms (T, t, u,

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v, w, U) may be expanded in Taylor series in terms of the distance from the wall,

$$S = S_0 + S_1 y + S_2 y^2 + S_3 y^3 + \ldots \quad (4)$$

where

$$S_n = \frac{1}{n!}\left(\frac{\partial^n S}{\partial y^n}\right)_{y=0} \quad (n = 1, 2, 3 \ldots). \quad (5)$$

From a no slip condition at the wall surface and from the mean and fluctuation velocity continuity equations, we know $v_1 = u_0 = v_0 = w_0 = U_0 = 0$ at $y = 0$, and $V = V_0 =$ const.

Substituting equation (4) into equation (3) and collecting coefficients in corresponding powers of y, one obtains

$$y^0: \quad \rho V_0 c_p T_1 = 2kT_2 + \mu[U_1^2 + \overline{u_1^2} + \overline{w_1^2}] \quad (6)$$

$$y^1: 2\rho V_0 c_p T_2 = 6kT_3 + \mu[4U_2 U_1 + \overline{4u_1 u_2} + \overline{4w_1 w_2}] \quad (7)$$

$$y^2: \quad 3\rho V_0 c_p T_3 = 12kT_4 + \mu\left[(4U_2^2 + 6U_1 U_3) + 12\overline{v_2^2}\right.$$
$$4\overline{v_2^2} + (4\overline{u_2^2} + \overline{6u_1 u_3}) + (4\overline{w_2^2} + \overline{6w_1 w_3}) + \overline{\left(\frac{\partial u_1}{\partial z}\right)^2}$$
$$\left. + \overline{\left(\frac{\partial u_1}{\partial x}\right)^2} + \overline{\left(\frac{\partial w_1}{\partial z}\right)^2} + \overline{\left(\frac{\partial w_1}{\partial x}\right)^2}\right] - 3\rho c_p \overline{t_1 v_2}. \quad (8)$$

Appropriate manipulations of equations (6–8) and the introduction of dimensionless nomenclature produce expressions for the temperature profile

$$T^+ = Pr\, y^+ + \left[\frac{Pr^2 V_0^+}{2!} - Pr\, N(1 + \overline{u_1^{+2}} + \overline{w_1^{+2}})\right] y^{+2}$$
$$+ \left[\frac{Pr^3 V_0^{+2}}{3!} - \frac{Pr^2 V_0^+ N}{3!}(1 + \overline{u_1^{+2}} + \overline{w_1^{+2}}) - \frac{2PrN}{3}(U_2^+ + \overline{w_1^+ w_2^+} + \overline{u_1^+ u_2^+})\right] y^{+3}$$
$$+ T_4^+ y^{+4} + \ldots, \quad (9)$$

and the turbulent heat flux,

$$\overline{v^+ t^+} = \overline{v_2^+ t_1^+}\, y^{+3} + \ldots, \quad (10)$$

where

$$A^+ = (Au^*/\nu); \quad (A = x, y \text{ or } z);$$
$$B^+ = (B/u^*); \quad (B = u, v, w, U);$$
$$C^+ = (C - T_w)\rho c_p u^*/\dot{q}_w, \quad (C = T \text{ or } t);$$

and

$$u^* = \sqrt{(\nu U_1)}; \quad N = -\rho u^{*3}/\dot{q}_w; \text{and } \dot{q}_w = -kT_1.$$

Physically N represents a ratio of viscous dissipation to heat flux near the wall, in terms of conventional nomenclature it may be expressed as

$$N = \frac{E_c}{C_H}\left(\frac{C_f}{2}\right)^{\frac{1}{2}};$$

i.e. a combination of the Eckert number, the Stanton number, and a conventional friction coefficient.

In [3] it was shown that $U_2^+ = \frac{1}{2}V_0^+$ and $U_3^+ = \frac{1}{6}V_0^{+2}$. The order of magnitude of $(u_1^+)^2$, $(w_1^+)^2$, $u_1^+ u_2^+$ and $w_1^+ w_2^+$ may be evaluated for the non-transpired case from measurements made in a pipe by Laufer [4]. These measurements suggest that the terms are all at least three orders of magnitude less than one; hence equation [9] may be approximated by

$$T^+ = Pr\, y^+ + \left[\frac{Pr^2 V_2^+}{2!} - \frac{PrN}{2!}\right](y^+)^2 + \left[\frac{Pr^3 V_0^{+2}}{3!} - \frac{Pr^2 V_0^+ N}{3!} - \frac{2PrNV_0^+}{3!}\right](y^+)^3 + T_4^+(y^+)^4 + \ldots. \quad (11)$$

When transpiration and dissipation are negligible, the functional relation for the temperature distribution near the wall reduces to that suggested by Tien [2, 5],

$$T^+ = Pr\, y^+ + T_4^+(y^+)^4 + \ldots. \quad (12)$$

Neglecting the higher order terms in the velocity fluctuations, the first term in the Taylor series expansion of the eddy heat flux in equation (10) is

$$\overline{v_2^+ t_1^+} \cong \left\{\frac{4T_4^+}{Pr} - \frac{Pr^3(V_0^+)^3}{12} + \frac{Pr^2(V_0^+)^2 N}{12} + \frac{Pr(V_0^+)^2 N}{3} + \frac{1}{3} N[2V_0^+ + (V_0^+)^2]\right\}. \quad (13)$$

The eddy diffusivities for heat and momentum may be written in dimensionless forms as

$$\left(\frac{\varepsilon_H}{\nu}\right) = -\overline{v^+ t^+}/(\mathrm{d}T^+/\mathrm{d}y^+),$$

and (14)

$$\frac{\varepsilon_M}{u} = -\overline{u^+ v^+}/(\mathrm{d}U^+/\mathrm{d}y^+).$$

For negligible dissipation and small transpiration it follows from equations (13, 14) and [3] that,

$$\frac{\varepsilon_H}{\nu} \simeq \left(\frac{Pr^2 V_0^+}{12} - \frac{4T_4^+}{Pr^2}\right)(y^+)^3 + \ldots,$$

and (15)

$$\frac{\varepsilon_M}{u} \simeq \left[\frac{(V_0^+)^3}{6} - 4U_4^+\right](y^+)^3 + \ldots.$$

Finally for no transpiration or dissipation very near the wall one might expect

$$\frac{\varepsilon_H}{\varepsilon_M} \simeq \frac{T_4^+}{U_4^+ Pr^2}, \quad (16)$$

where T_4^+ is also a function of Prandtl's number.

The constant U_4^+ may be obtained by matching the Taylor series expansion for the velocity distribution near the wall to empirical expressions for the velocity further from the wall [1, 3]. Unfortunately, no universal relation appears to exist for the turbulent temperature distribution as yet. However, equation (16) suggests $(\varepsilon_H/\varepsilon_M)$ is at most a function of the Prandtl number for the simplest case [5].

CONCLUSIONS

It is observed that the eddy diffusivity of heat varies with the cubic power of the wall distance with or without inclusion of transpiration and dissipation effects. This disagrees with the assumption of Deissler that the eddy diffusivity varies as the square of the distance at low Prandtl numbers and with the fourth power at high Prandtl numbers [2]. However, it is in agreement with the very successful analysis presented by Lin *et al.* that the diffusivity goes as the third power near the wall. The analytical formulations of Lin *et al.* correlated heat and mass transport data over a Prandtl or Schmidt number range from 0·54 to 3200. This note provides analytical justification for their empirical usage of an eddy diffusivity coefficient proportional to the third power of the wall distance.

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HEAT TRANSFER IN THERMAL ENTRANCE REGION OF COCURRENT FLOW HEAT EXCHANGERS WITH FULLY DEVELOPED LAMINAR FLOW

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NOMENCLATURE

A, dimensionless velocity gradient at the wall between streams, defined below equation (6);
a_k, coefficients of expansion in the wall temperature distribution, defined below equation (15);
C_p, heat capacity;
D_h, equivalent diameter of the annulus;
H, capacity ratio, $\dfrac{w_2 C_{p2}}{w_1 C_{p1}}$;
K, conductivity ratio, k_1/k_2;
k, thermal conductivity;
Nu, Nusselt number, defined in equations (16) and (17);
Pe, Péclet number, $u_B D_h/\alpha$;
r, radial coordinate;
r_0, r_1, r_2, r_3, radii defined on Fig. 1;
r^*, dimensionless radius ratio, defined on Fig. 1;
T, temperature;
u, axial velocity;
w, mass flow rate;
x, axial coordinate;
$\bar{x}$, dimensionless axial coordinate, $4(x/D_{h_1})/Pe_1$;
$\bar{x}_2$, $4(x/D_{h_2})/Pe_2$;
$\bar{y}$, dimensionless radial coordinate, defined below equation (6).

Greek symbols

α, thermal diffusivity;
η, similarity variable, defined below equation (6);
θ, dimensionless temperature, $\dfrac{T_i - T_{e_1}}{T_w - T_{e_1}}$;